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Partial privatization in mixed duopoly

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Abstract

We investigate a quantity-setting duopoly involving a private firm and a privatized firm jointly owned by the public and private sectors. The private firm maximizes profits, while the privatized firm takes both profits and social welfare into consideration. We consider how many shares the government should hold in the privatized firm. We find that neither full privatization (the government does not hold any shares) nor full nationalization (the government holds all of the shares) is optimal under moderate conditions.

Keywords: Quantity-setting duopoly; Partial privatization; Mixed oligopoly

JEL classification: C72; H42; L13

1. Introduction

Recently a literature on 'mixed markets', involving private and public enterprises, has began to appear.[1] In fact, in many countries a lot of public firms compete with private firms in private goods markets.[2] Many existing works assume that the public firm maximizes social welfare (the sum of consumer's surplus and profits by firms) while the private firm maximizes its own profits. De Fraja and Delbono (1989) show that welfare may be higher when a public firm is a

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[1] See Vickers and Yarrow (1988); De Fraja and Delbono (1990); Bös (1991) for surveys. See also Harris and Wiens, 1980; Cremer et al., 1989; Estrin and de Meza, 1995.

[2] In Japan, in particular, we observe competition between private and public firms in many oligopolistic markets. See Yoshino and Fujita (1995); Hayashi and Ide (1992).

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profit-maximizer rather than a welfare-maximizer. This result suggests that in some cases a public firm should be privatized and should maximize profits rather than welfare.[3] However, De Fraja and Delbono (1989) (as well as most existing works) neglect the possibility of partial privatization. In many cases the government has usually held or even still holds a non-negligible proportion of shares in privatized firms.

With the exception of the USA, we can observe many firms with a mixture of private and public ownership. Since privatized firms with mixed ownership must respect the interests of private shareholders, they cannot be pure welfare-maximizers. At the same time they must respect the interests of the government, so they cannot be pure profit-maximizers. By controlling the shares that the government holds, it may be able to indirectly control the activities of the privatized firm. In such situations it is important to consider the proportion of shares in privatized firms the government should hold.

In this paper we explicitly allow the possibility of partial privatization and investigate how many shares in the privatized firm the government should hold. We try to highlight a reason why the government sells part but not all of its shares in public firms in the context of 'mixed oligopoly'. We assume that a privatized firm maximizes a weighted average of the payoff of the government and its own profits, and that the weight is affected by the proportion of shares held by the government. We find that full nationalization (the government holds all shares in the firm) is not optimal unless the public firm is a monopolist. This result suggests that in the context of mixed oligopoly the public firm should be (at least partially) privatized. Furthermore, we find that full privatization (the government sells all shares in the public firm) is not optimal if the public firm is as efficient as the private firm. These results suggest that partial privatization is a reasonable choice for the government in the context of mixed oligopoly.

The remainder of this paper is organized as follows. In Section 2, we formulate a model. In Section 3 we discuss the optimal level of shares held by the government. Section 4 concludes the paper.

2. The model

The duopolists produce perfectly substitutable commodities for which the market demand function is given by $p(q)$: $\Re_+ \to \Re_+$ (price as a function of quantity).

Firm 1 is a privatized firm which is jointly owned by both public and private

[3] One of the main purposes of privatizing a public firm is to improve its productivity, but this effect is not considered by De Fraja and Delbono (1989). One of the contributions of De Fraja and Delbono (1989) is to show that privatization of a public firm is beneficial even if the privatization does not reduce the production cost of the firm.

sectors, and firm 2 is a pure private firm. The public sector owns $s \in [0,1]$ shares in firm 1. Firm 2 is assumed to maximize its profits, while firm 1 is assumed to maximize the weighted average of payoff of the government and its own profit.[4] The social welfare W is the sum of consumer's surplus and profits by both firms, and is given by

$$W(x_1, x_2) = \int_0^X p(q)\mathrm{d}q - pX + \Pi_1(x_1, x_2) + \Pi_2(x_2, x_1)$$
$$= \int_0^X p(q)\mathrm{d}q - c_1(x_1) - c_2(x_2),$$

where Π_i $(i=1,2)$ is firm i's profit, x_i $(i=1,2)$ is firm i's output quantity and $X \equiv x_1 + x_2$.

The government's payoff U_G is given by

$$U_G(x_1, x_2) = W(x_1, x_2) + \beta\left(\int_0^X p(q)\mathrm{d}q - pX\right)$$

where β is a constant. If β is zero, the government wants to maximize the social welfare. If β is positive, the government respects the consumer's surplus rather than profits. We assume that the government respects consumer's surplus at least as highly as profits.[5]

Assumption 1. $\beta \geq 0$.

Firm i's cost function is given by $c_i(x_i)$: $\Re_+ \to \Re_+$. Firm 1's payoff U_1 and firm 2's payoff U_2 are respectively given by

$$U_1 = \alpha U_G(x_1, x_2) + (1-\alpha)\Pi_1(x_1, x_2),\ U_2 = \Pi_2(x_2, x_1), \tag{1}$$

where $\alpha (\in [0,1])$ is the weight of the payoff of the government for firm 1's objection.

We assume that the government can indirectly control α through its shareholding. In other words we assume that α depends on s. If firm 1 is fully

[4]In this paper we do not allow the government to nationalize both firms. As pointed out by Merrill and Schneider (1966) the most efficient outcome is achieved by the nationalization of both firms in the case where nationalization does not change the costs of firms. The need for an analysis of mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without a competitor public firms may lose the incentive to improve their costs; resulting in a loss of social welfare. Thus we neglect the possibility of nationalizing both firms.

[5]If β is negative, Proposition 1(i) does not hold true. This implies that in mixed duopoly 100% ownership may be efficient if the government respects profits rather than consumer's surplus.

privatized (i.e., $s=0$), α becomes zero. If firm 1 is fully nationalized (i.e., $s=1$), α becomes one.[6] If the shares owned by the government increase, then α increases.[7] Formally we make the following assumptions.

Assumption 2. $\alpha(s)$ is continuous, non-decreasing, $\alpha(0)=0$, and $\alpha(1)=1$.[8]

Assumption 3. $c_i(x_i)$ $(i=1,2)$ is twice differentiable $\forall x_i>0$, and $p(q)$ is twice differentiable $\forall p>0$ and $q\geq 0$;

Assumption 4. c_i $(i=1,2)$ is strictly increasing $\forall x_i\geq 0$, and $p'(q)<0$ $\forall q\geq 0$ and $p>0$.

Note that Assumptions 3 and 4 allow the discontinuity of the cost function $c_i(x_i)$ at $x_i=0$.

We now describe the model. The game is a complete information game. Before the game, s is given exogenously and is observed by each firm. Thus each firm observes α, since $\alpha(s)$ is known. Each firm i $(i=1,2)$ independently maximizes U_i with respect to x_i $(\in \Re_+)$ given its rival's output x_j $(j=1,2,\ j\neq i)$. The first order conditions for firm 1 and 2 are respectively

$$(1-\alpha)p'x_1 + p - c_1' - \alpha\beta p'X = 0; \text{ and} \tag{2}$$

$$p'x_2 + p - c_2' = 0. \tag{3}$$

Assumption 5. The relevant second order conditions for Eqs. (2) and (3) are satisfied.

Definition 1 (Reaction functions). For firm 1 and firm 2 we define

$$R_1(x_2\colon \alpha, \beta) \equiv \arg\max_{x_1\geq 0} U_1(x_1, x_2\colon \alpha, \beta), \text{ and } R_2(x_1) \equiv \arg\max_{x_2\geq 0} U_2(x_2, x_1)$$

Assumption 6. $-1\leq \partial R_1/\partial x_2$ (or $c''\geq(1-\alpha)p'$), $-1<\partial R_2/\partial x_1<0$ (or $c''\geq p'$ and $p''x_2+p'<0$).

Definition 2 (Equilibrium output). Let $E_1(\alpha, \beta)$ and $E_2(\alpha, \beta)$ denote the equilibrium output of the above game. They are given by $E_1=R_1(E_2)$ and $E_2=R_2(E_1)$.

[6]Even a pure public firm might not maximize the payoff of the government due to incentive problems regarding the management of the firm. This simplifying assumption separates the issue of the principal-agent problem from the analysis of mixed oligopolies. See De Fraja and Delbono (1990).

[7]For a rationalization of this objective function for the partially privatized firm, see Bös (1991) ch. 8.

[8]We can replace the assumption of continuity of $\alpha(s)$ with the following without changing any results: There exist $\bar{\alpha}\in(0,1)$ and $\underline{\alpha}\in(0,1)$ which satisfy the following two conditions: (1) for any $x\in[0, \underline{\alpha}]$ there exists s such that $\alpha(s)=x$; and (2) for any $x\in[\underline{\alpha}, 1]$ there exists s such that $\alpha(s)=x$.

Result 1. Suppose that Assumptions 1–6 are satisfied. Suppose that $E_1 > 0$ and $E_2 > 0$. Then E_1 is increasing in α and E_2 is decreasing in α.

Proof. See Appendix A.

The intuition behind Result 1 is obvious. Under Assumption 1, the government has a stronger incentive to increase the output than the private sector. This is the reason why E_1 is increasing in α. Since R_2 is decreasing in x_1, the above aggressive behavior of firm 1 reduces the equilibrium output of firm 2. This is the reason why E_2 is decreasing in α.

Since the equilibrium outcome depends on α and β, social welfare and the payoff of the government also depend on α.

Definition 3. Define the equilibrium welfare by $W^E(\alpha, \beta) \equiv W(E_1(\alpha, \beta), E_2(\alpha, \beta))$, and define the equilibrium payoff of the government by $U_G^E(\alpha, \beta) \equiv U_G(E_1(\alpha, \beta), E_2(\alpha, \beta))$.

3. Results

We now discuss the optimal level of s. Proposition 1 (i) (resp. (ii)) states that the optimal level of s for the social welfare (resp. the government) is strictly smaller than one as long as $E_1(1, \beta), E_2(1, \beta) > 0$.

Proposition 1. *Suppose that Assumptions* 1–6 *are satisfied. Suppose that* $E_1(1, \beta) > 0$. *Then*

(i). $1 \in \arg\max_{\{s \in [0,1]\}} W^E(\alpha(s), \beta)$ *only if* $E_2(1, \beta) = 0$, *and*
(ii). $1 \in \arg\max_{\{s \in [0,1]\}} U_G^E(\alpha(s), \beta)$ *only if* $E_2(1, \beta) = 0$,

Proof. See Appendix A.

Proposition 1 states that $s = 1$ is optimal only if a private firm (firm 2) cannot enter the market. In other words, if a public firm is not a monopolist, the government should (at least partially) privatize the public firm.

Proposition 1 (i) and (ii) are exactly the same when the government wants to maximize the social welfare (i.e., β is zero). We now explain the intuition of the result in this case. Assumption 2 guarantees that a decrease in s reduces α. As is shown by Result 1, the decrease in α increases firm 2's output and reduces firm 1's output. If s is one, α is one by Assumption 2. If α is one, firm 1 is a welfare-maximizer, so the price is equal to firm 1's marginal cost. Thus a slight reduction in x_1 does not reduce W (i.e., $\partial W / \partial x_1 = p - c_1' = 0$). On the other hand, firm 2 is a profit-maximizer, so the price is strictly larger than firm 2's marginal cost. Thus an increase in x_2 improves W (i.e., $\partial W/\partial x_2 = p - c_2' > 0$). Therefore, a slight reduction of α from one improves social welfare. Thus $s=1$ ($\alpha=1$) is never optimal. If β is positive, firm 1 has a larger incentive to expand its output quantity, resulting in a more significant loss of social welfare.

Proposition 1 implies that pure welfare-maximizing behavior of firm 1 is harmful in mixed duopoly, but it does not state that pure profit-maximizing behavior is optimal. In fact, the optimal level of α depends on demand and cost conditions, and it is difficult to derive any general properties. However, we obtain a clear result when firm 1 and firm 2 have the same cost function.

Proposition 2. *Suppose that* $c_1(x)=c_2(x)\ \forall x$. *Suppose that Assumptions* 1–6 *are satisfied. Suppose that the equilibrium is symmetric when* $\alpha=0$. *Then* (i) $0 \notin \arg\max_{\{s\in[0,1]\}} W^{E}(\alpha(s),\beta)$, *and* (ii) $0 \notin \arg\max_{\{s\in[0,1]\}} U_{G}^{E}(\alpha(s),\beta)$.

Proof. See Appendix A.

Proposition 2 states that full privatization is not optimal if the two firms have the same cost function. Proposition 2 holds true if firm 1 is strictly more efficient than firm 2, but it does not always hold true if firm 1 is strictly less efficient than firm 2.[9]

We now present an example showing that whether or not the full privatization is optimal depends on the cost conditions facing the two firms. We assume that $\beta=0$.

We consider the case of constant marginal costs. Let $mc_i (i=1,2)$ denote the marginal cost of firm i. We assume that $mc_1 > mc_2 \geq 0$. Suppose that $p(q)$ is given by $p=a-b(x_1+x_2)$ where a and b are positive constants. In the linear demand and linear cost cases above, the optimal level of s is positive if and only if $1/5 > (mc_1 - mc_2)/a$.

We now interpret this result. Suppose that firm 1 is fully privatized. Then full privatization is optimal only if the market share of firm 1 (i.e., $E_1(0,0)/(E_1(0,0)+E_2(0,0))$) is smaller than $1/3$. Suppose that firm 1 is a public firm (i.e., $s=1$). Then full privatization is optimal only if the market share of firm 1 (i.e., $E_1(1,0)/(E_1(1,0)+E_2(1,0))$) is small than $3/4$.

Propositions 1 and 2 do not compare the welfare in the mixed duopoly where both firms produce with that of the monopoly by firm 1. Some readers may think that, even if pure-welfare maximizing behavior by firm 1 does not deter firm 2 from entering the market, the monopoly by firm 1 may be desirable when the entry cost of firm 2 is high. In this case, if public firm can commit to pricing below marginal cost, this may be desirable as a way of pre-empting wasteful entry. We now briefly consider this problem by using the example above. We introduce the entry cost of firm 2. Let F_2 denote the entry cost. We find that the monopoly by firm 1 is desirable if and only if $F_2 > \frac{3}{2b}(mc_1 - mc_2)^2$. Firm 1 can deter the entry

[9] Following discussion is closely related with Lahiri and Ono (1988) which discussed asymmetric Cournot duopoly.

of firm 1 by pure welfare-maximizing behavior if and only if $F_2 > \frac{1}{b}(mc_1 - mc_2)^2$. Since $\frac{3}{2b}(mc_1 - mc_2)^2 > \frac{1}{b}(mc_1 - mc_2)^2$, we have that firm 1 can always deter firm 2 from entering by marginal cost pricing when the monopoly by firm 1 is desirable. This result, however, is not robust. In some cases of declining marginal costs, pricing below marginal cost is desirable as a way of pre-empting wasteful entry.

4. Concluding remarks

In this paper we investigate a quantity-setting duopoly involving a private firm and a privatized firm with mixed ownership between public and private sectors. We consider the problem of how many shares the government should hold in the privatized firm. We show that the government should not hold all of the shares in the privatized firm. We also find that full privatization is not optimal if the two firms have the same cost function.

We now want to emphasize that this paper might underestimate the benefit of full privatization of public firms. The costs of public firms may be higher than those of private firms. Reducing α may improve the performance of the privatized firm. In this paper we neglect the above possible benefit of privatization.

Finally, we must admit that our model is restrictive. For example our model does not allow the government to have the objection to maximize consumer's surplus subject to minimum profit of the public firm. In this case 100% ownership of public firm is optimal, so Proposition 1 does not hold true. We do not consider the solvency problem of the public firm. Public firms are often given the objective of maximizing output subject to breaking even. Moreover, given the problem of maximizing welfare subject to a breakeven constraint, it will often be the case that the constraint binds. In this case 100% ownership of the public firm is optimal, so Proposition 1 does not hold true. Furthermore, we do not allow the discontinuity of reaction function which may induce multiple equilibria. Extending our model to these direction remains for future research.

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Appendix A

Proof of Result 1

We consider the case where E_1 and E_2 are positive. Note that $E_1(\alpha, \beta)$ and $E_2(\alpha, \beta)$ are given by Eqs. (2) and (3) as long as E_1 and E_2 are positive.

From Eqs. (2) and (3), we have that

$$\frac{dE_1}{d\alpha} = \frac{(p'x_1 + \beta p'X)(p''x_2 + 2p' - c_2'')}{G+H}. \tag{4}$$

$$\frac{dE_2}{d\alpha} = -\frac{(p'x_1 + \beta p'X)(p''x_2 + p')}{G+H}. \tag{5}$$

where $G \equiv ((1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X + p'))(p''x_2 + 2p' - c_2'')$ and $H \equiv -((1-\alpha)p''x_1 + p'(1-\alpha) - \alpha\beta(p''X+p'))(p''x_2 2p')$.

$$(1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X + p') < 0;\text{ and} \tag{6}$$

$$p''x_2 + 2p' - c_2'' < 0. \tag{7}$$

From Eqs. (6) and (7) we have that $G > 0$.

From Eqs. (2) and (3) we have that

$$\begin{aligned} \frac{\partial R_1}{\partial x_2} &= -\frac{(1-\alpha)p''x_1 + p' - \alpha\beta(p''X+p')}{(1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X+p')}; \\ \frac{\partial R_2}{\partial x_1} &= -\frac{p''x_2 + p'}{p''x_2 + 2p' - c_2''}. \end{aligned} \tag{8}$$

From Assumption 6 and Eq. (8), we have that

$$-1 \leq -\frac{(1-\alpha)p''x_1 + p' - \alpha\beta(p''X+p')}{(1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X+p')};\ \text{and} \tag{9}$$

$$-1 < -\frac{p''x_2 + p'}{p''x_2 + 2p' - c_2''} < 0. \tag{10}$$

From Eq. (7) and the second inequality in Eq. (10), we have that

$$p''x_2 + p' < 0. \tag{11}$$

From the inequalities in Eq. (10), we have that $|p''x_2 + 2p' - c_2''| > |p''x_2 + \mathrm{p}'|$. From Eqs. (9) and (6), we have that $|(1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X+p')| \geq |(1-\alpha)p''x_1 + p' - \alpha\beta(p''X+p')|$ (when $R_1' < 0$) or $(1-\alpha)p''x_1 + p' - \alpha\beta(p''X+p') \geq 0$ (when $R_1' \geq 0$.) If $|(1-\alpha)p''x_1 + (2-\alpha)p' - c_1'' - \alpha\beta(p''X+p')| \geq |(1-\alpha)p''x_1 + p'(1-\alpha) - \alpha\beta(p''X+p')|$, we have that $|G| \geq |H|$. Note that $0 \leq \alpha \leq 1$ and $p' < 0$.

Since G is positive, we have that $G+H$ is positive. If $(1-\alpha)p''x_1+p'-\alpha\beta(p''X+p')\geq 0$, from Eq. (11), we have that $H\geq 0$. Therefore in both cases $G+H$ is positive.

Since $p'<0$, $\beta\geq 0$ and $G+H>0$, from Eqs. (4) and (7) we have that $\partial E_1/\partial\alpha>0$, and from Eq. (5) and Eq. (11) we have that $\partial E_2/\partial\alpha<0$. Q.E.D.

Proof of proposition 1(i)

We show that the optimal level of s is not one if $E_1(1, \beta)$, $E_2(1,\beta)>0$. Under Assumption 2 the government can choose any $\alpha\in[0,1]$. Assumption 2 ensures that the optimal level of s is one if and only if the optimal level of α is one. Thus, all we have to show is that the optimal level of α is not one. We will prove this by showing that $\mathrm{d}W^{\mathrm{E}}(1,\beta)/d\alpha$ is negative.

From the definitions of W^{E}, E_1, E_2, and R_2, we have that

$$W^{\mathrm{E}}(\alpha,\beta)=\int_0^{E_1(\alpha,\beta)+R_2(E_1(\alpha,\beta))} p(q)\mathrm{d}q-c_1(E_1(\alpha,\beta))-c_2(R_2(E_1(\alpha,\beta)). \tag{12}$$

From Result 1, we have that $\mathrm{d}E_1/\mathrm{d}\alpha>0$. Since $\mathrm{d}W^{\mathrm{E}}/\mathrm{d}\alpha=(\mathrm{d}W^{\mathrm{E}}/\mathrm{d}E_1)(\mathrm{d}E_1/d\alpha)$, we have that $\mathrm{d}W^{\mathrm{E}}/\mathrm{d}\alpha<0$ if and only if $\mathrm{d}W^{\mathrm{E}}/\mathrm{d}E_1<0$.

From Eq. (12), we have that

$$\frac{\mathrm{d}W^{\mathrm{E}}}{\mathrm{d}E_1}=p-c_1'+R_2'(p-c_2') \tag{13}$$

From Eq. (2) we have that $p-c_1'=\beta p'(E_1+R_2(E_1))$ when $\alpha=1$. From Assumptions 1 and 4, we have that $\beta p'(E_1+R_2(E_1))\leq 0$.

We now show that $R_2'(p-c_2')<0$. From Assumptions 4, 6 and Eq. (3) we have that this inequality is satisfied. Note that Eq. (3) is satisfied if $E_2(1, \beta)$ is positive. Q.E.D.

Proof of Proposition 1(ii)

We show that the optimal level of s for the government is not one if $E_1(1, \beta)$, $E_2(1, \beta)>0$. Under assumption 2 the government can choose any $\alpha\in[0,1]$. Assumption 2 ensures that the optimal level of s is one if and only if the optimal level of α is one. Thus, all we have to show is that the optimal level of α is not one. We will prove this by showing that $\mathrm{d}U_{\mathrm{G}}^{\mathrm{E}}(1)/\mathrm{d}\alpha$ is negative.

From the definitions of $U_{\mathrm{G}}^{\mathrm{E}}$, E_1, E_2, and R_2, $U_{\mathrm{G}}^{\mathrm{E}}(\alpha, \beta)$ is given by

$$U_{\mathrm{G}}^{\mathrm{E}}(\alpha,\beta)=(1+\beta)\int_0^X p(q)\mathrm{d}q-c_1-c_2-\beta pX, \tag{14}$$

where $X\ E_1(\alpha,\ \beta)+R_2(E_1(\alpha,\ \beta))$. From Result 1, we have that $\mathrm{d}E_1/\mathrm{d}\alpha>0$. Since $\mathrm{d}U_G^E/\mathrm{d}\alpha=(\mathrm{d}U_G^E/\mathrm{d}E_1)(\mathrm{d}E_1/\mathrm{d}\alpha)$, we have that $\mathrm{d}U_G^E/\mathrm{d}\alpha<0$ if and only if $\mathrm{d}U_G^E/\mathrm{d}E_1<0$.

From Eq. (14), we have that

$$\frac{\mathrm{d}U_G^E}{\mathrm{d}E_1}=p-c_1'-\beta p'(E_1+R_2(E_1))+R_2'(p-c_2'-\beta p'(E_1+R_2(E_1))). \quad (15)$$

From Eq. (2) we have that $p(E_1+R_2(E_1))-c_1'-\beta p'(E_1+R_2(E_1))=0$ when $\alpha=1$. Note that Eq. (2) is satisfied if $E_1(1,\ \beta)$ is strictly positive. From Assumptions 1, 4, 6 and Eq. (3) we have that $R_2'(p-c_2'-\beta p'(E_1+R_2(E_1)))<0$. Thus, Eq. (15) is negative. Q.E.D.

Proof of proposition 2(i)

We show that the optimal level of s is not zero if the two firms have the same cost function. Assumption 2 ensures that the optimal level of s is zero if and only if the optimal level of α is zero. Thus, all we have to show is that the optimal level of α is not zero. We will prove this by showing that $\mathrm{d}W^E(0,\ \beta)/\mathrm{d}\alpha$ is positive.

When α is zero, the equilibrium is symmetric. Thus, $c_1'=c_2'$ in equilibrium. From Assumption 6, we have that $p-c_1'+R_2'\ (p-c_2')>0$. From Eq. (13), we have that $\mathrm{d}W^E\ (0,\ \beta)/d\alpha$ is positive. Q.E.D.

Proof of proposition 2(ii)

We will prove this Proposition by showing that $\mathrm{d}U_G^E\ (0,\ \beta)/\mathrm{d}\alpha$ is positive. Since $c_1'=c_2'$, from Assumption 1, 4, and 6 we have that $p-c_1'-\beta p'(E_1+R_2(E_1))+R_2'(p-c_2'-\beta p'(E_1+R_2(E_1)))>0$. From Eq. (15), we have that $\mathrm{d}W^E\ (0,\ \beta)/d\alpha$ is positive. Q.E.D.

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