第七章 影响线及其应用

一 影响线的作用与概念

二 利用影响线求量值

三 某量值最不利荷载位置

四 简支梁的绝对最大弯矩

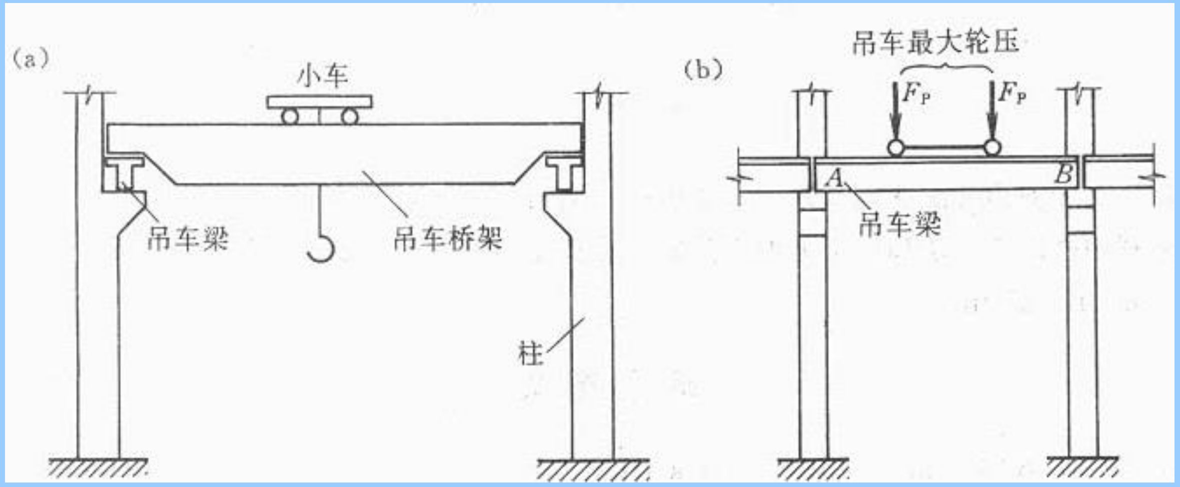
五 简支梁的内力包络图

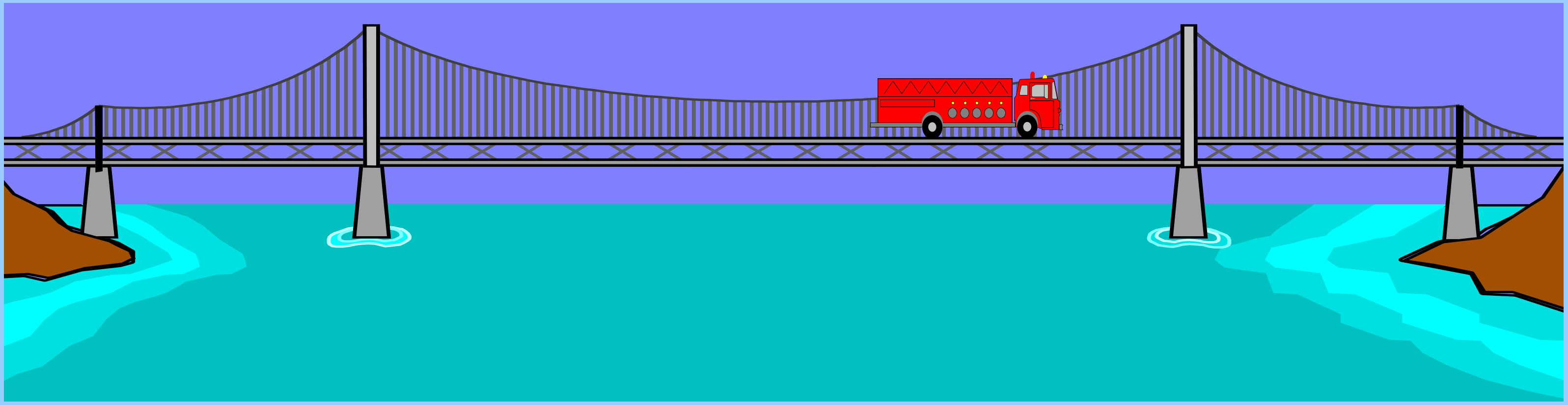
六 超静定梁在可动均布荷载下的最不利荷载

一 影响线的作用与概念

解决**移动荷载**作用下结构的内力计算问题。

移动荷载：大小、方向、间距维持不变、作用位置发生变化的一组荷载.



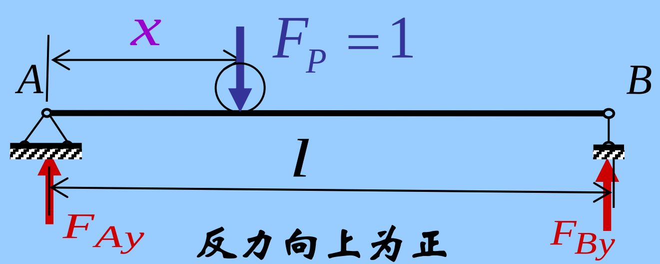


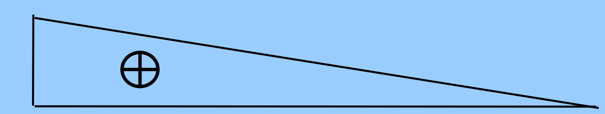
定义：当一个方向不变的单位荷载沿一结构移动时，表示某指定截面的某一量值变化规律的函数图形，称为该量值的影响线。

作影响线的方法有:静力法和机动法

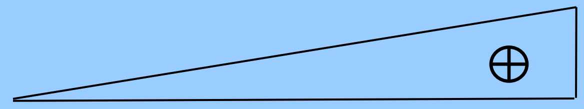
1. 静力法作影响线

(1)简支梁的影响线





F_{Ay}影响线



F_{By}影响线

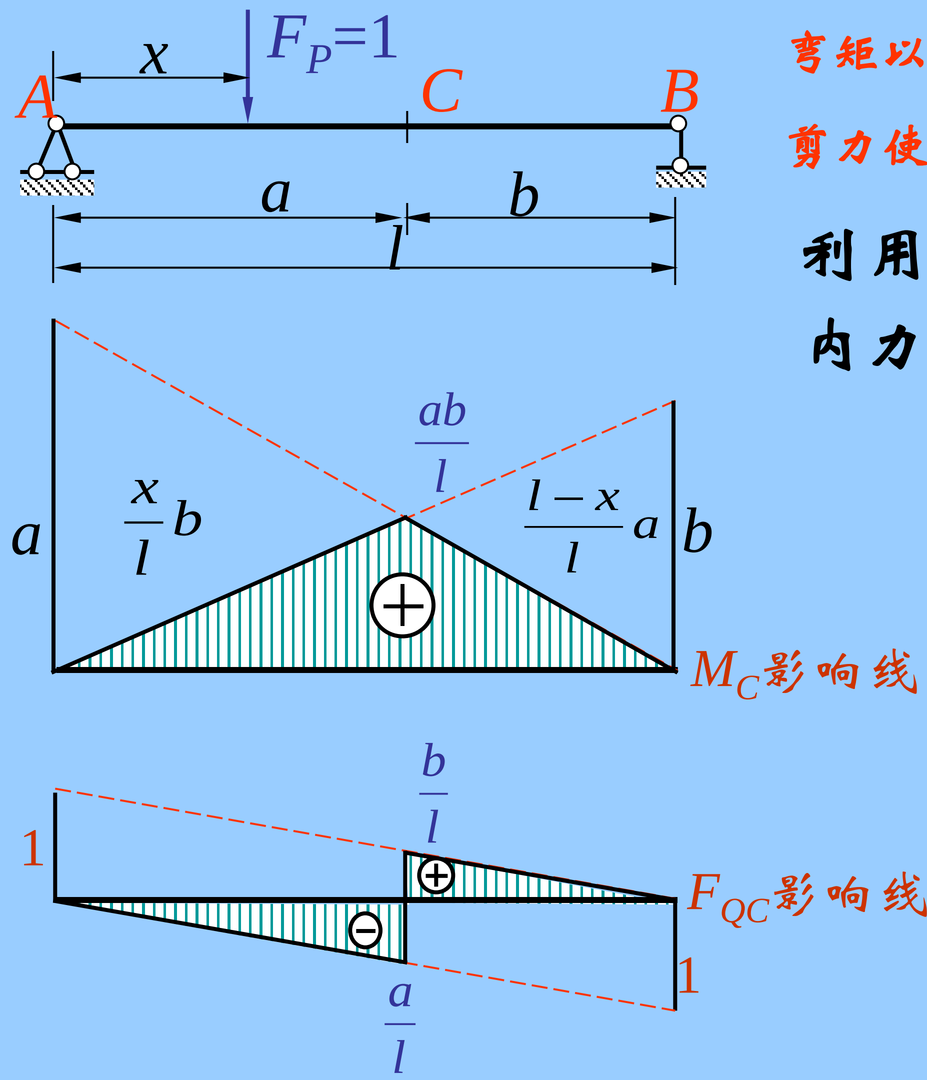
①反力影响线

利用**平衡条件**建立反力影响线方程：

$$F_{Ay} = \frac{l-x}{l} \left(0 \le x \le l\right)$$

$$F_{By} = \frac{x}{l} \left(0 \le x \le l\right)$$

①内力影响线



弯矩以使梁下边纤维受拉为正

剪力使隔离体顺时针转动为正

利用**平衡条件**建立

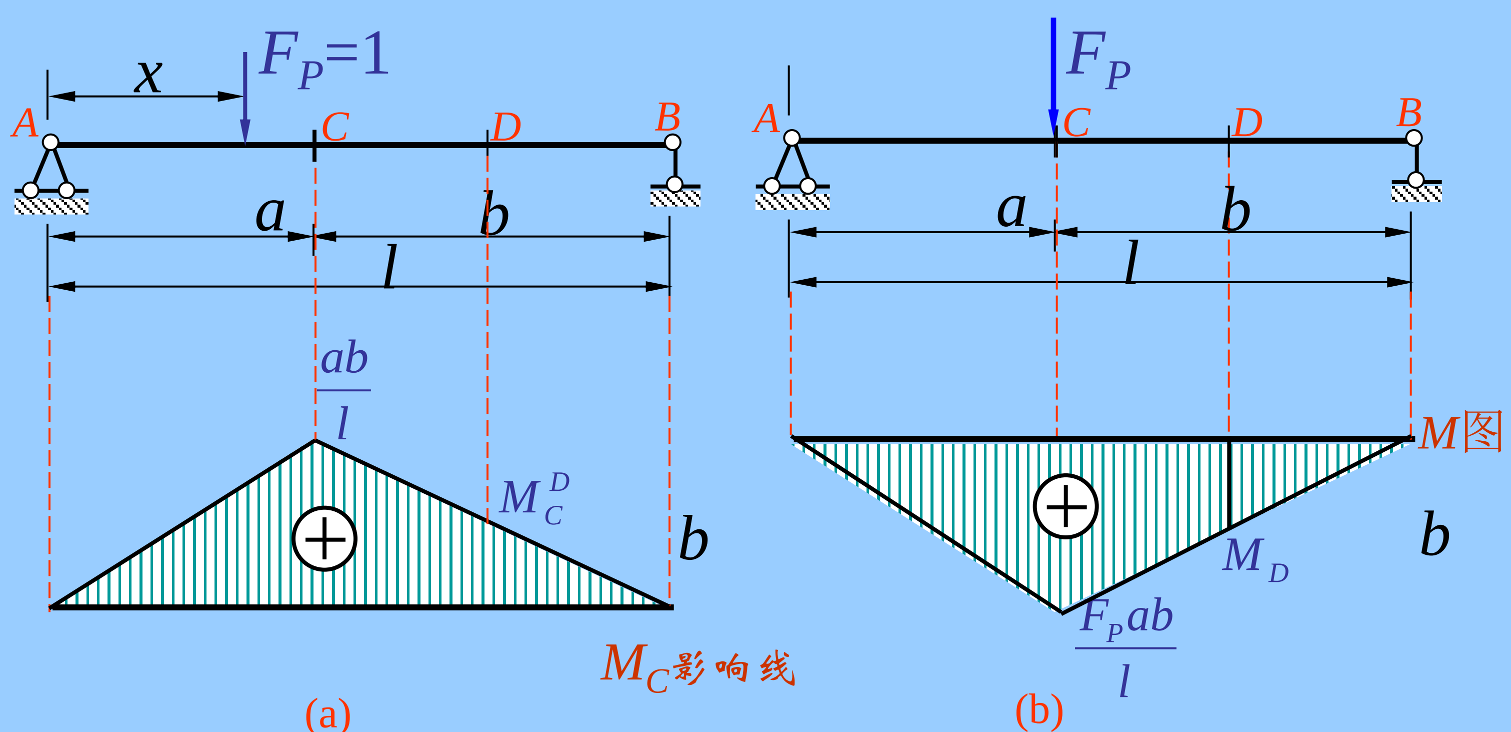
内力影响线方程:

$$M_C=\begin{cases}\frac{x}{l}b & 0\le x\le a\\ \frac{l-x}{l}a & a\le x\le l\end{cases}$$

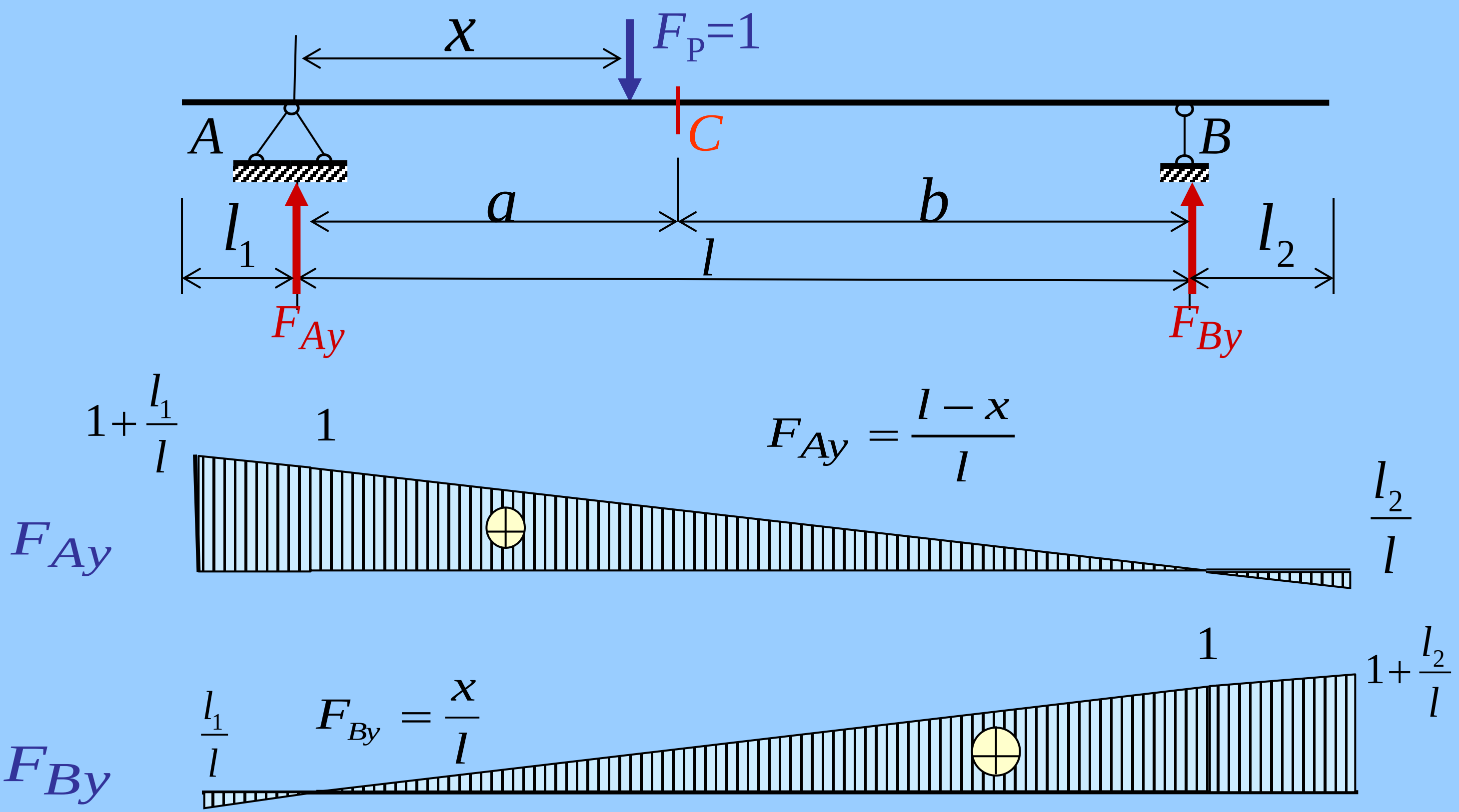
$$F_{QC}=\begin{cases}-\frac{x}{l} & 0\le x\le a\\ \frac{l-x}{l} & a\le x\le l\end{cases}$$

影响线与内力图的比较

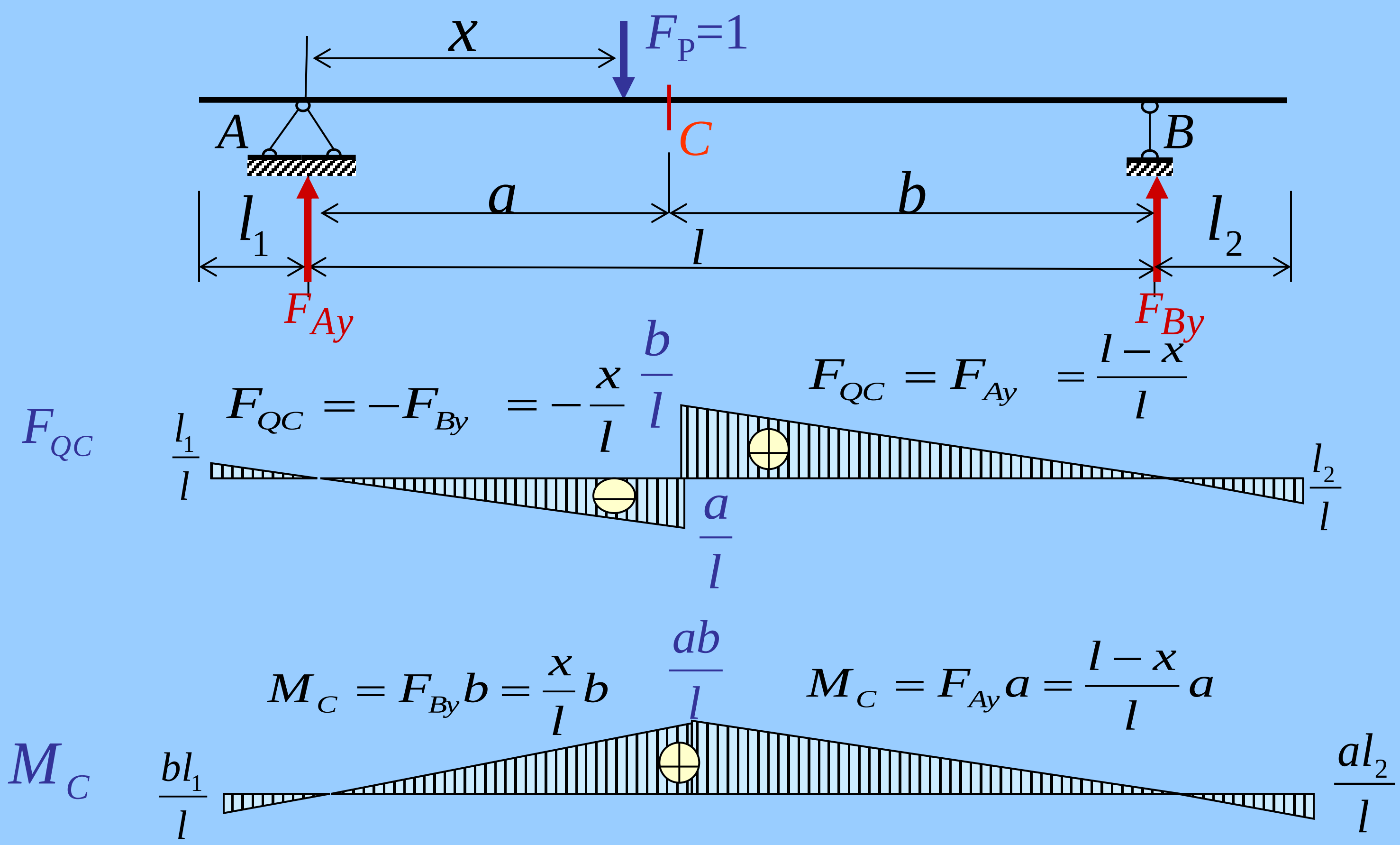
	影响线	内力图
荷载大小	1	实际
荷载性质	移动	固定
横座标	表示荷载位置	表示截面位置
纵座标	表示某一截面内力变化规律	表示全部截面内力分布规律



(2)伸臂梁的影响线　①反力影响线

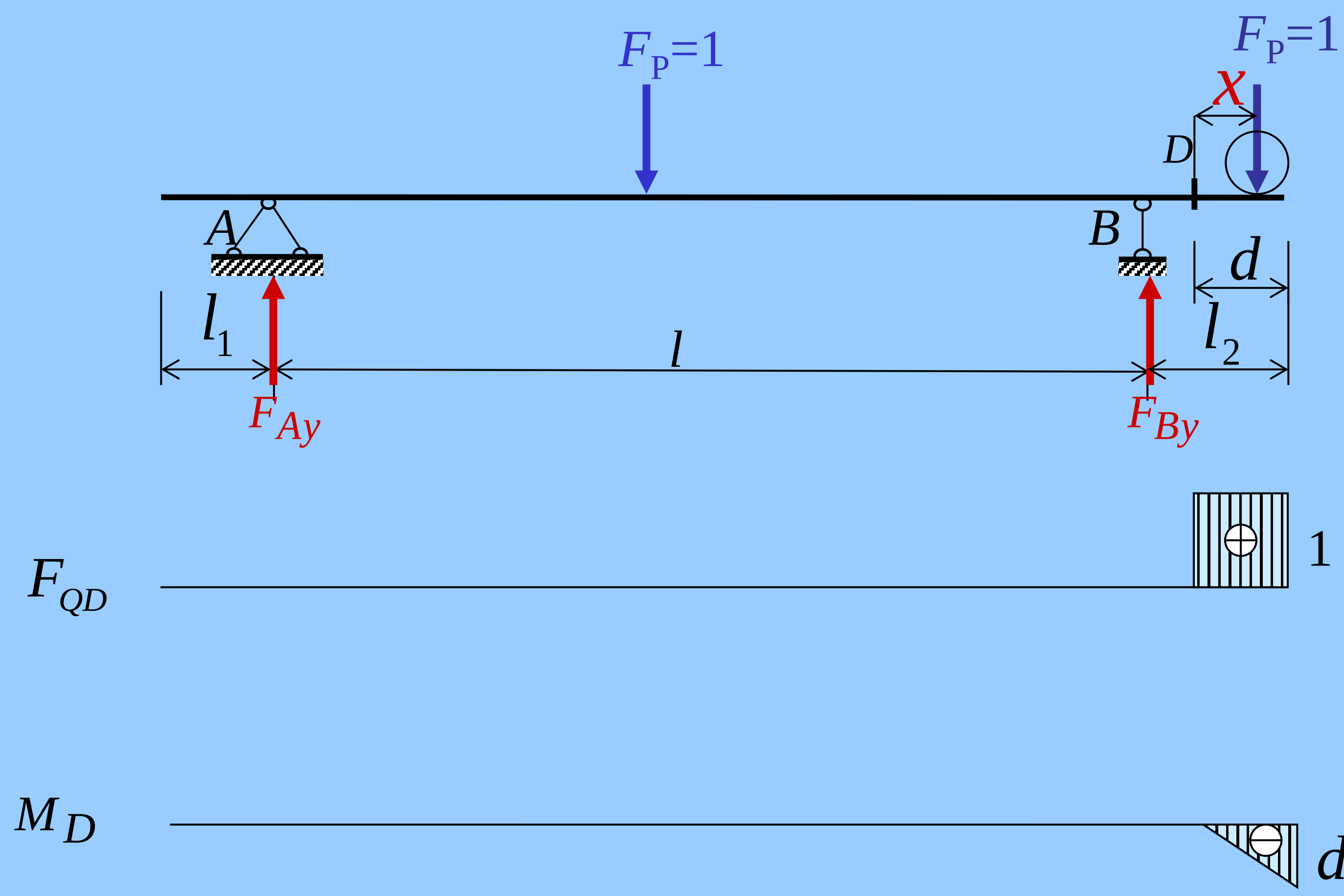


②内力影响线



x
$F_P=1$
A
C
B
l_1
a
b
l
l_2
F_{Ay}
F_{By}
F_{QC}
$\frac{l_1}{l}$
$F_{QC}=-F_{By}=-\frac{x}{l}$
$\frac{b}{l}$
$F_{QC}=F_{Ay}=\frac{l-x}{l}$
$\frac{l_2}{l}$
$\frac{a}{l}$
M_C
$M_C=F_{By}b=\frac{x}{l}b$
$\frac{ab}{l}$
$M_C=F_{Ay}a=\frac{l-x}{l}a$
$\frac{bl_1}{l}$
$\frac{al_2}{l}$

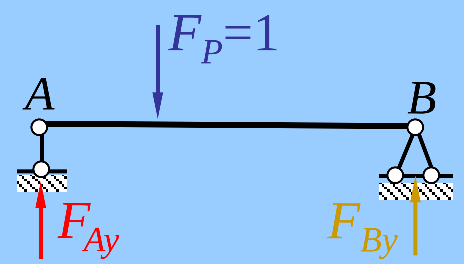
伸臂梁的伸臂部分截面D的内力影响线



2. 机动法作影响线

理论基础：刚体虚功方程

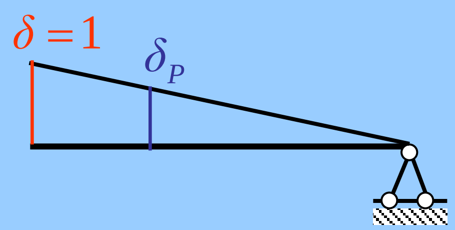
实际力状态



$$W=\sum F_P\cdot\Delta=0$$

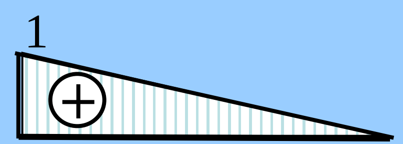
$$F_{Ay}\delta-F_P\delta_P=0$$

虚设位移状态



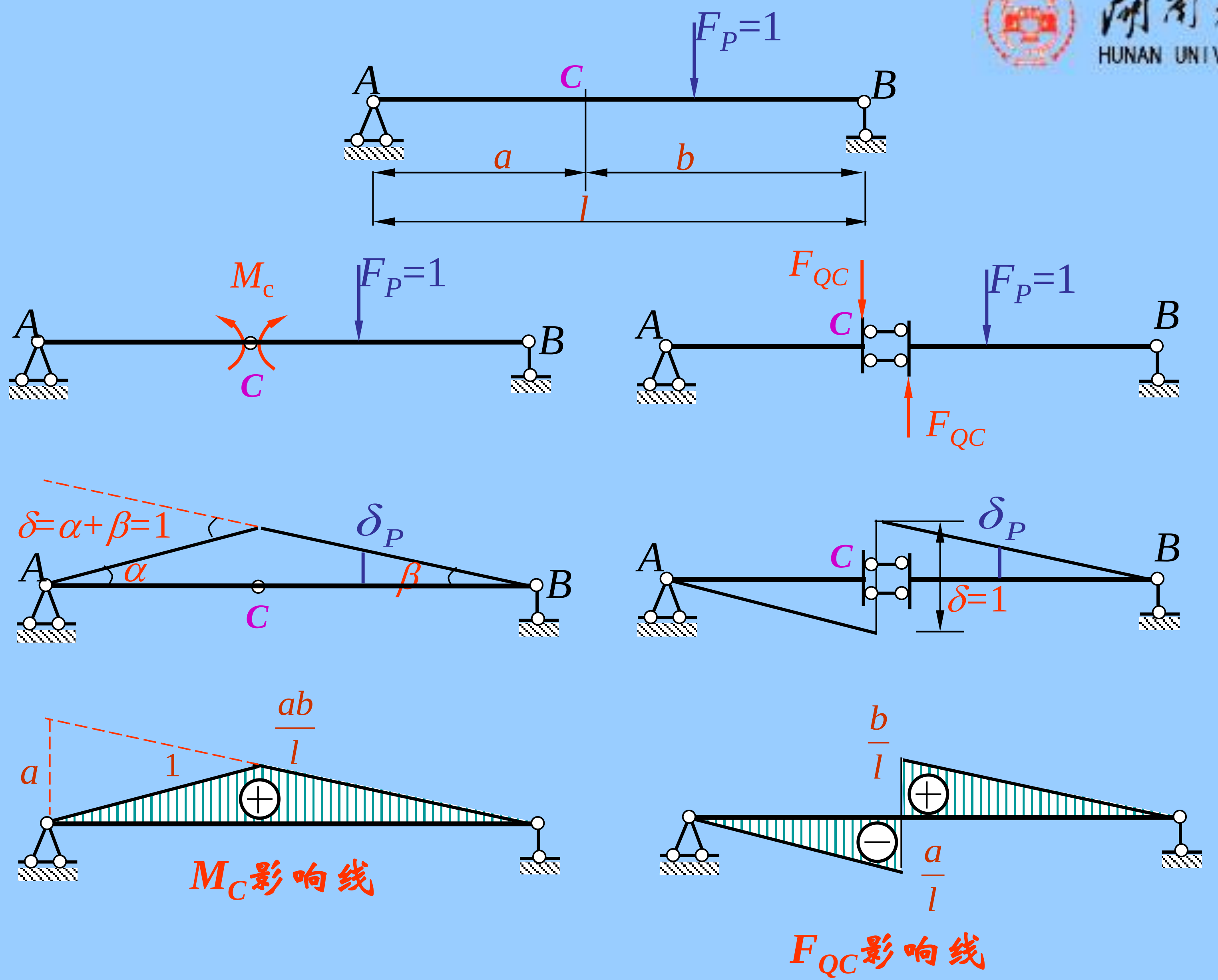
$$F_{Ay}=\delta_P$$

F_{Ay}影响线



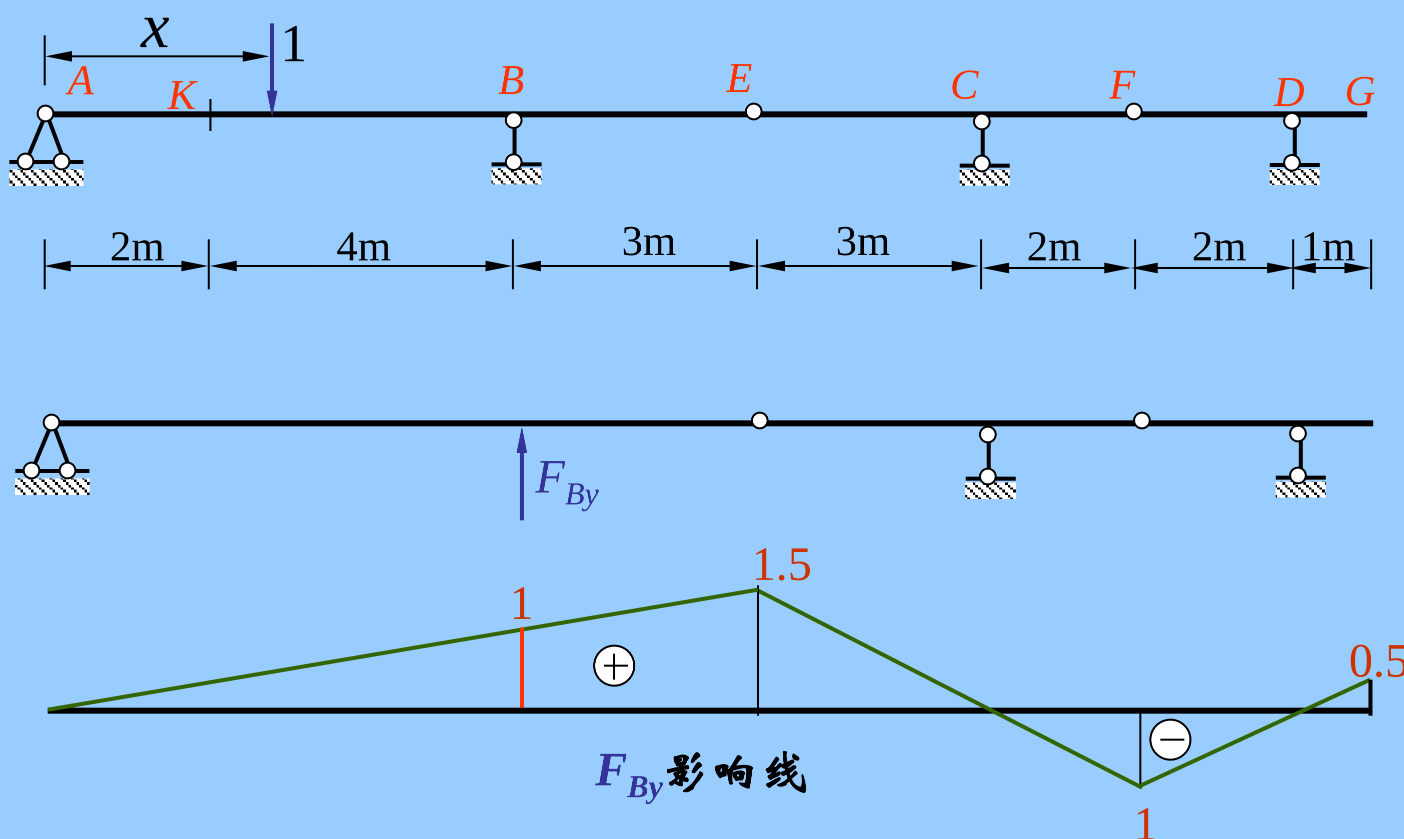
δ_P 大小随F_P移动变化.

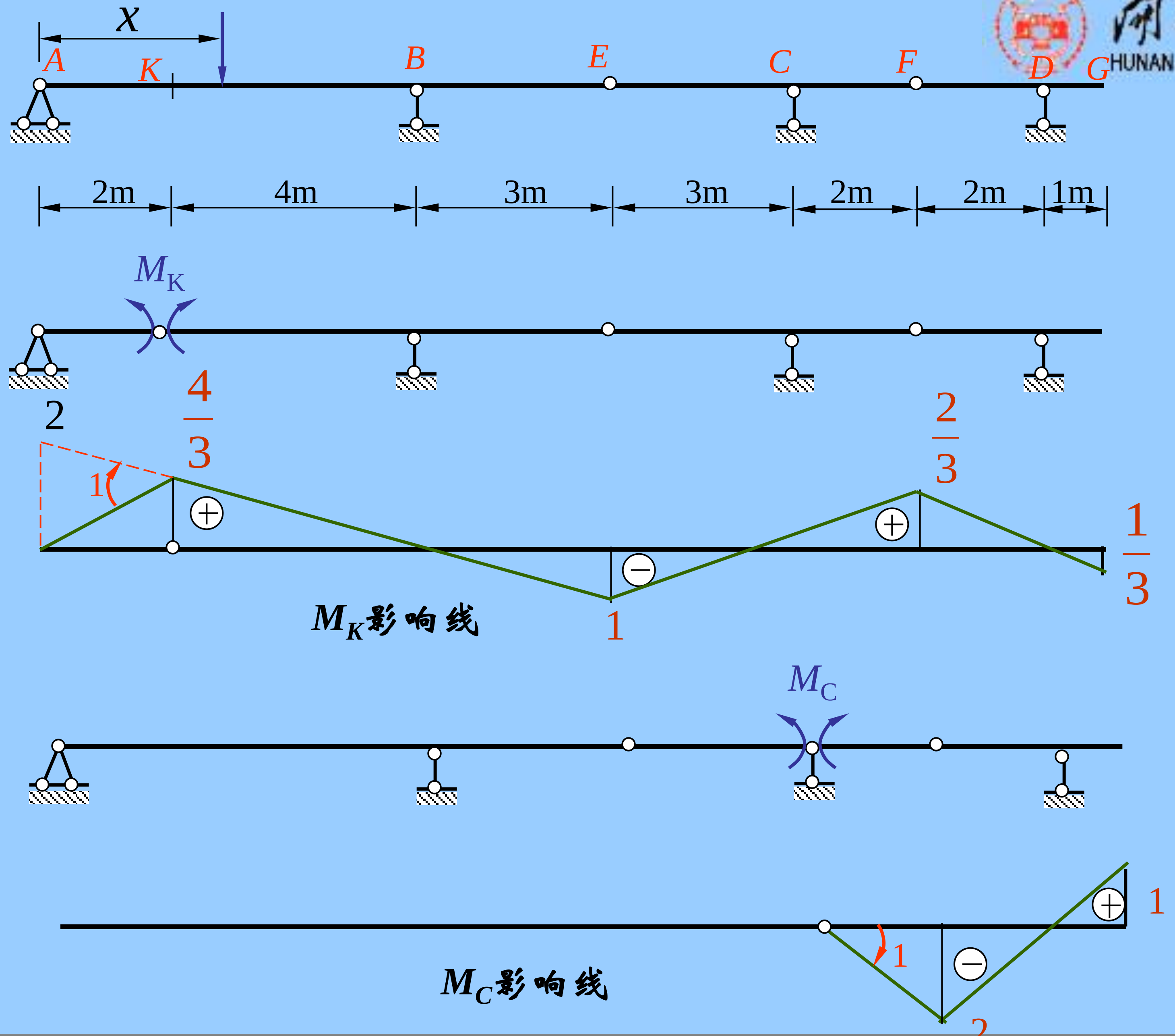
将与F_{Ay}相应的约束去掉，使所得机构沿其正方向发生单位位移，所得虚位移图即为该反力的影响线.



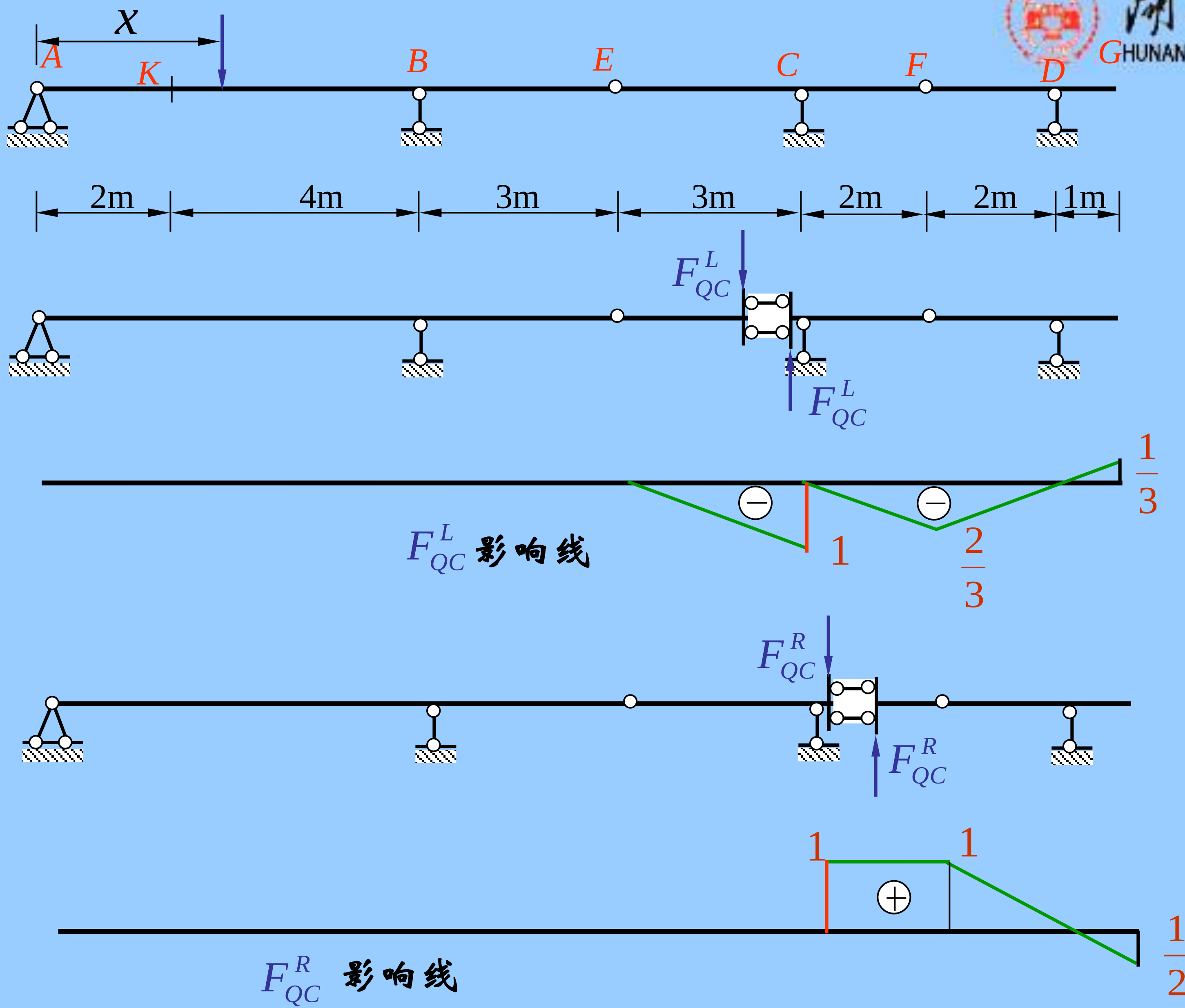
A
C
B
$F_P=1$
a
b
l
M_c
F_{QC}
$\delta=\alpha+\beta=1$
α
β
δ_P
$\delta=1$
ab/l
a
1
b/l
a/l
M_C影响线
F_{QC}影响线

例 用机动法绘制所示多跨静定梁的 $F_{By}, M_K, M_C, F_{QC}^L, F_{QC}^R$ 影响线。



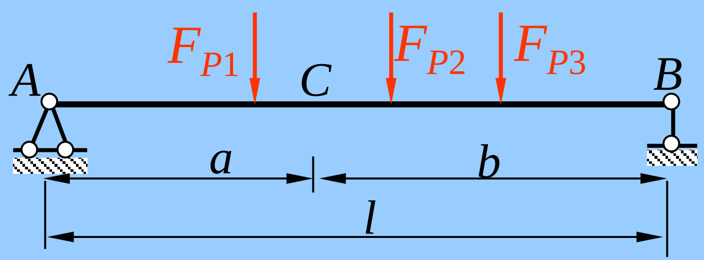


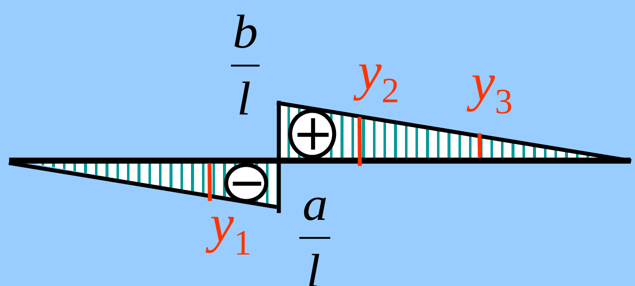
x
A
K
B
E
C
F
D
G
2m
4m
3m
3m
2m
2m
1m
M_K
2
4/3
1
2/3
1/3
1
M_K影响线
M_C
1
1
2
M_C影响线


x
A
K
B
E
C
F
D
2m
4m
3m
3m
2m
2m
1m
F_{QC}^{L}
F_{QC}^{L}
1
1/3
2/3
F_{QC}^{L} 影响线
F_{QC}^{R}
F_{QC}^{R}
1
1
1/2
F_{QC}^{R} 影响线

二 利用影响线求影响量

1.受集中力作用时





F_{QC}影响线

$$F_{QC} = F_{P1}y_1 + F_{P2}y_2 + F_{P3}y_3$$

推广到一般,有

$$S = \sum_{i=1}^{n} F_{Pi}y_i$$

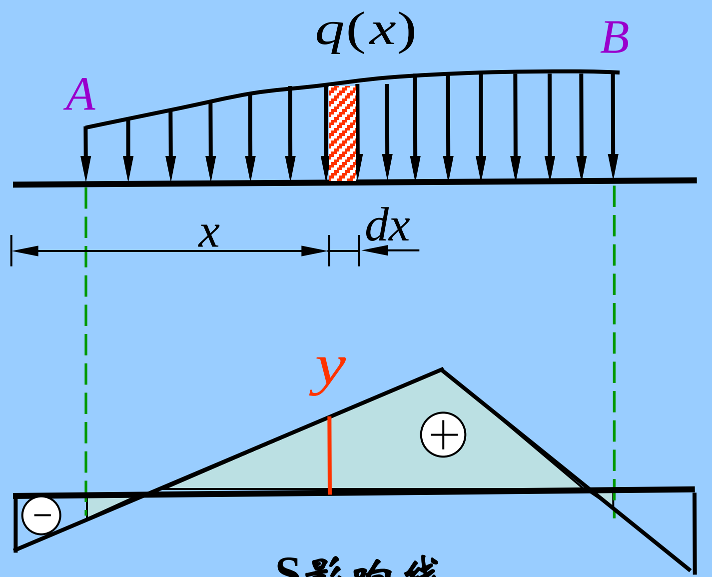
注意：影响线竖标y_i的正、负号。

特殊情况:当荷载作用在影响线的某段直线部分时:

$$S = \sum_{i=1}^{n} F_{Pi}y_i = tg\theta.\sum F_{Pi}x_i$$

$$= tg\theta.F.\bar{x} = F.\bar{y}$$

2. 受分布荷载作用时



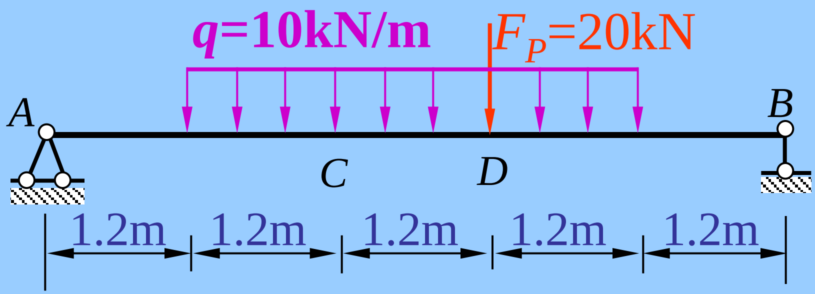
S影响线

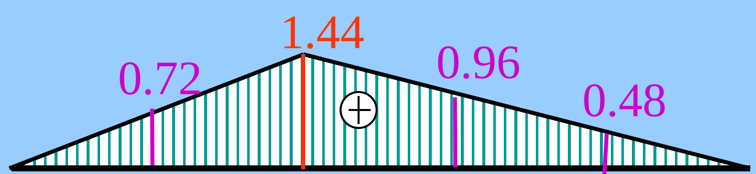
$$S = \int_A^B q(x) y dx$$

特殊情况:

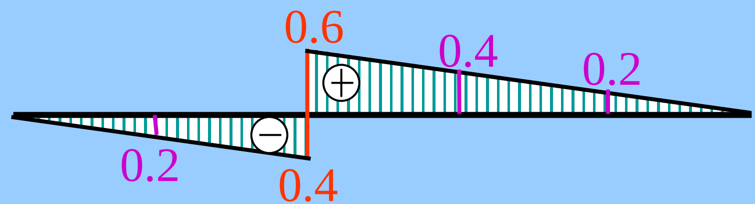
$$S = q\int_A^B y dx = qA$$

例 试利用影响线求简支梁在图示荷载作用下M_C和F_{QC}的值。





M_C影响线(单位m)



F_{QC}影响线

$$M_C = 20\times 0.96 + 10\times[\frac{1}{2}\times(1.44+0.72)\times 1.2 + \frac{1}{2}\times(1.44+0.48)\times 2.4] = 55.2\text{kN}\cdot\text{m}$$

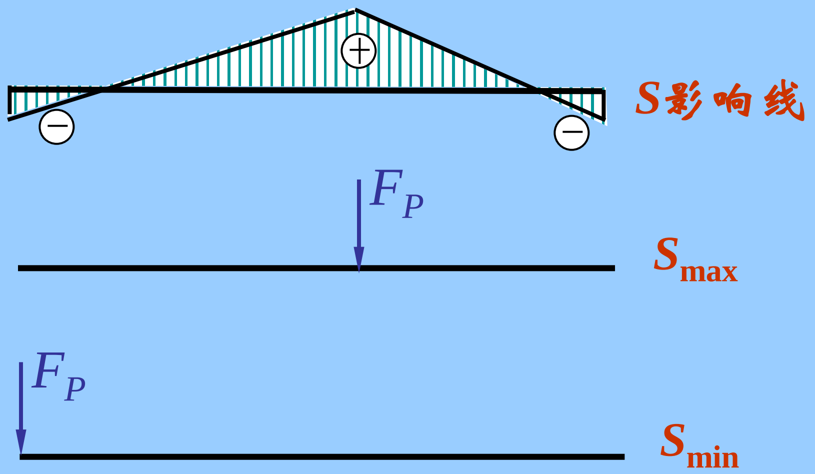
（下部受拉）

$$F_{QC} = 20\times 0.4 + 10\times[\frac{1}{2}\times(0.6+0.2)\times 2.4 - \frac{1}{2}\times(0.2+0.4)\times 1.2] = 14\text{kN}$$

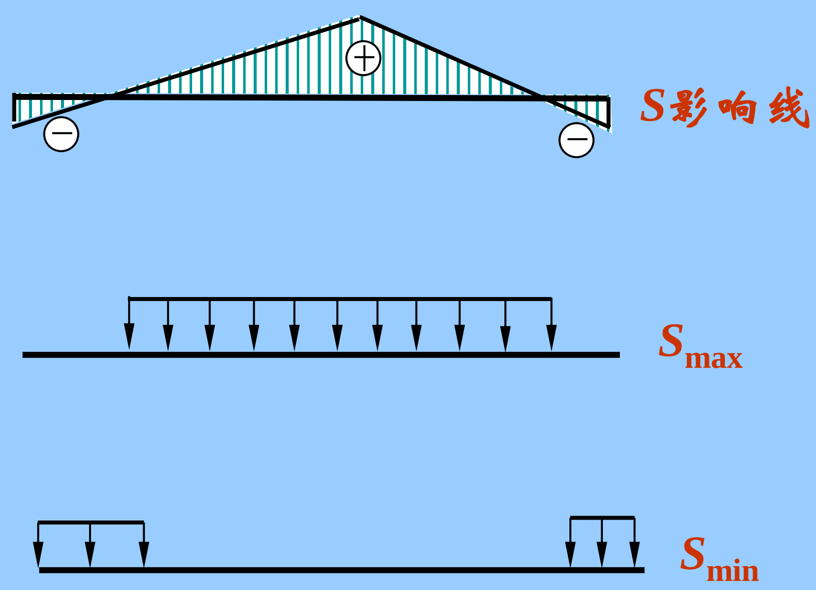
三 某量值最不利荷载位置

如果移动荷载移到某个位置，使某量值达到**最大正值S_{max}或最大负值S_{min}**，则此荷载位置称为该量值S的最不利荷载位置。

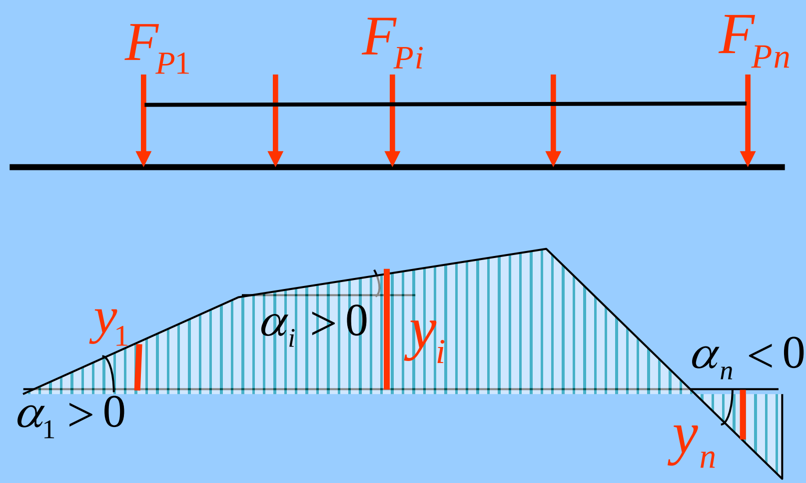
1.一个可动集中荷载情形



2.可动均布荷载(分布宽度及位置可以任意指定)情形



3.行列荷载



S影响线

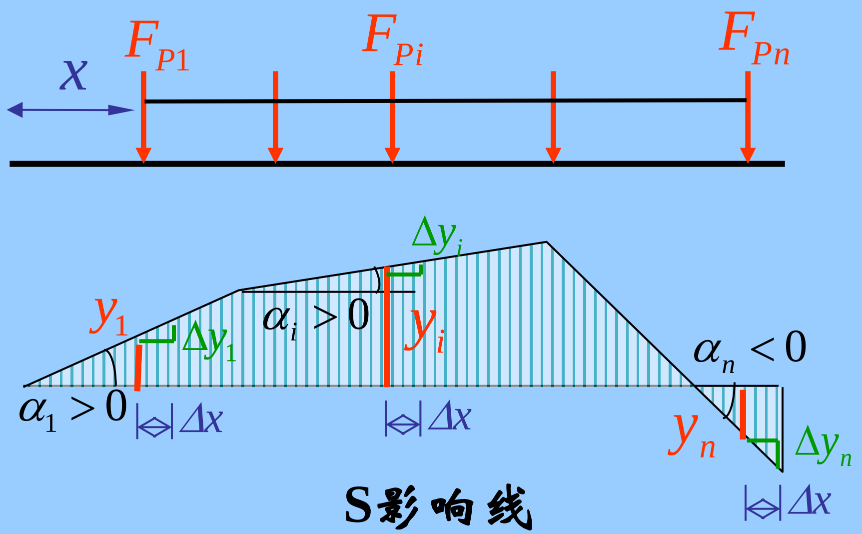
$$S(x) = \sum_{i=1}^{n} F_{Pi} \cdot y_i$$

取极大值条件

$$\begin{cases} \dfrac{dS}{dx} = 0 \\ \dfrac{d^2S}{dx^2} < 0 \end{cases} \Longleftrightarrow \begin{cases} \dfrac{dS}{dx} \mid x^L \geq 0 \\ \dfrac{dS}{dx} \mid x^R \leq 0 \end{cases}$$

取极小值条件

$$\begin{cases} \dfrac{dS}{dx} = 0 \\ \dfrac{d^2S}{dx^2} > 0 \end{cases} \Longleftrightarrow \begin{cases} \dfrac{dS}{dx} \mid x^L \leq 0 \\ \dfrac{dS}{dx} \mid x^R \geq 0 \end{cases}$$



$$S(x) = \sum_{i=1}^{n} F_{Pi} \cdot y_i$$

$$S(x+\Delta x) = \sum_{i=1}^{n} F_{Pi} \cdot (y_i + \Delta y_i) = S(x) + \sum_{i=1}^{n} F_{Pi} \cdot \Delta y_i$$

$$\Delta S = \sum F_{Pi} \cdot \Delta y_i = \sum F_{Pi} \cdot \Delta x \cdot \tan\alpha_i = \Delta x \sum F_{Pi} \cdot \tan\alpha_i$$

$$\frac{\Delta S}{\Delta x} = \sum F_{Pi} \cdot \tan\alpha_i$$

$$\frac{\Delta S}{\Delta x}=\sum F_{Pi}\cdot\tan\alpha_i$$

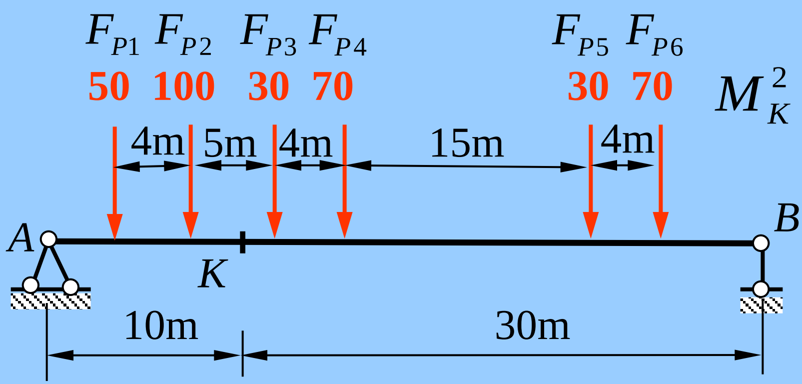
极值位置时只要荷载移动使$\sum F_{Pi}\cdot\tan\alpha_i$变号。

至少有一个F_{Pi}对应的α_i有变化。

使$\sum F_{Pi}\cdot\tan\alpha_i$变号的荷载称为临界荷载，用$F_{\mathrm{pcr}}$表示.

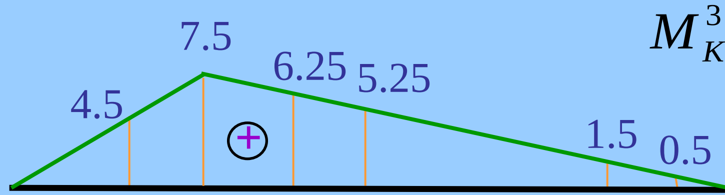
$\sum F_{Pi}\cdot\tan\alpha_i$称为临界荷载判别式.

例 试求图示简支梁在图示移动荷载作用下**截面K的最大弯矩**。荷载单位：kN



当 F_{P2}＝100kN在截面K时

$$M_K^2 = 50\times4.5+100\times7.5+30\times(6.25+1.5)$$
$$+70\times(5.25+0.5)=1610\text{kN}\cdot\text{m}$$

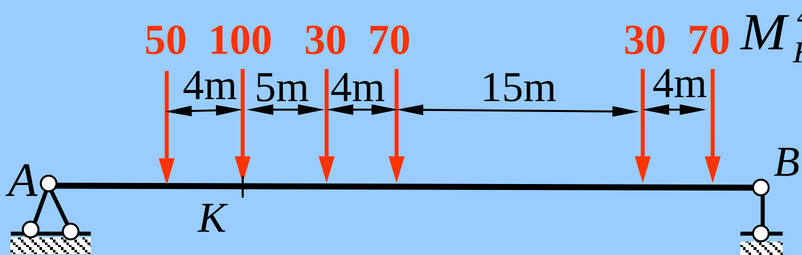


M_K影响线

当 F_{P3}＝30kN在截面K

$$M_K^3 = 50\times0.75+100\times3.75+30\times(7.5+2.75)$$
$$+70\times(6.5+1.75)=1297.5\text{kN}\cdot\text{m}$$

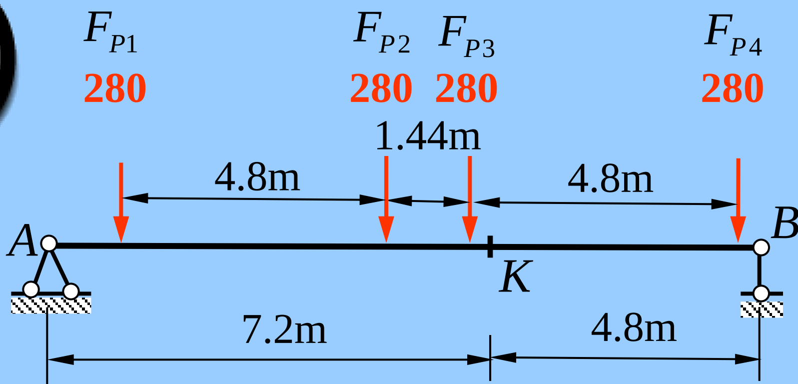
当 F_{P4}＝70kN在截面K

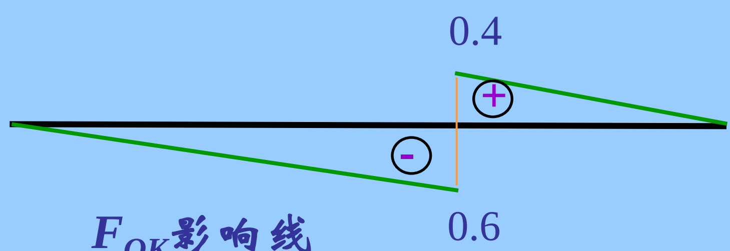


$$M_K^4 = 100\times0.75+30\times(4.5+3.75)$$
$$+70\times(7.5+2.75)=1040\text{kN}\cdot\text{m}$$

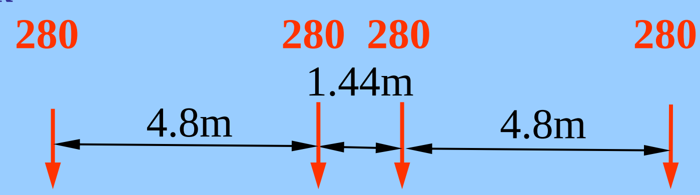
$$M_{K\max} = M_K^2 = 1610\text{kN}\cdot\text{m}$$

例 试求图示简支梁在图示移动荷载作用下截面K的最大剪力 F_{QKmax} 和最小剪力 F_{QKmin}。荷载单位：kN

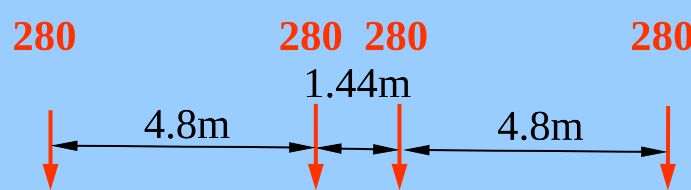




F_{QK}影响线



F_{QK}（max）



F_{QK}（min）

$$F_{QK(\max)} = 280\times(0.4+0.28-0.2)$$
$$=134.4\text{kN}$$

$$F_{QK(\min)} = 280\times(-0.6-0.48-0.08)$$
$$=-324.8\text{kN}$$

四 简支梁的绝对最大弯矩

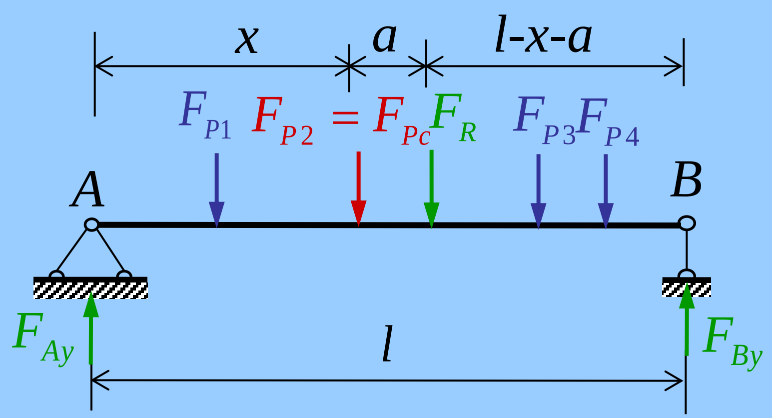
梁的绝对最大弯矩：

在一组移动集中荷载作用下,梁的各截面最大弯矩中的最大者.

要确定梁的绝对最大弯矩,须确定:

一是确定绝对最大弯矩所在截面;

二是确定对应的最不利荷载位置.



$$\sum M_B = 0, F_{Ay} = F_R \cdot \frac{l-x-a}{l}$$

$$M_2 = F_R \cdot \frac{l-x-a}{l} \cdot x - M_{left}$$

$$\frac{dM}{dx} = 0, \frac{F_R}{l}(l-2x-a) = 0$$

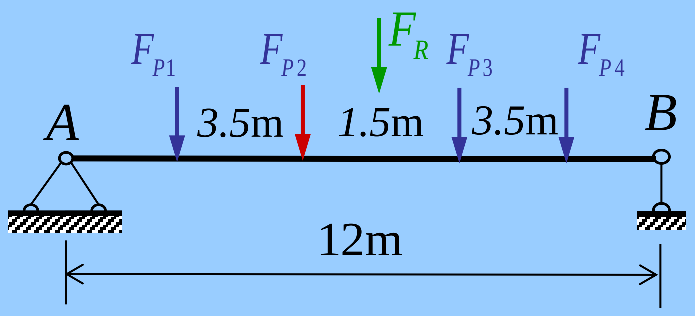
$$x = \frac{l-a}{2} \qquad M_{2\max} = F_R\left(\frac{l-a}{2}\right)^2 \frac{1}{l} - M_{left}$$

以上过程可以对其它F_{Pi}试算

$$x=\frac{l-a}{2}$$

$$M_{2\max}=F_R\left(\frac{l-a}{2}\right)^2\frac{1}{l}-M_{left}$$

$$F_{P1}=F_{P2}=F_{P3}=F_{P4}=82\text{kN}$$



$$a=0.75\text{m}$$

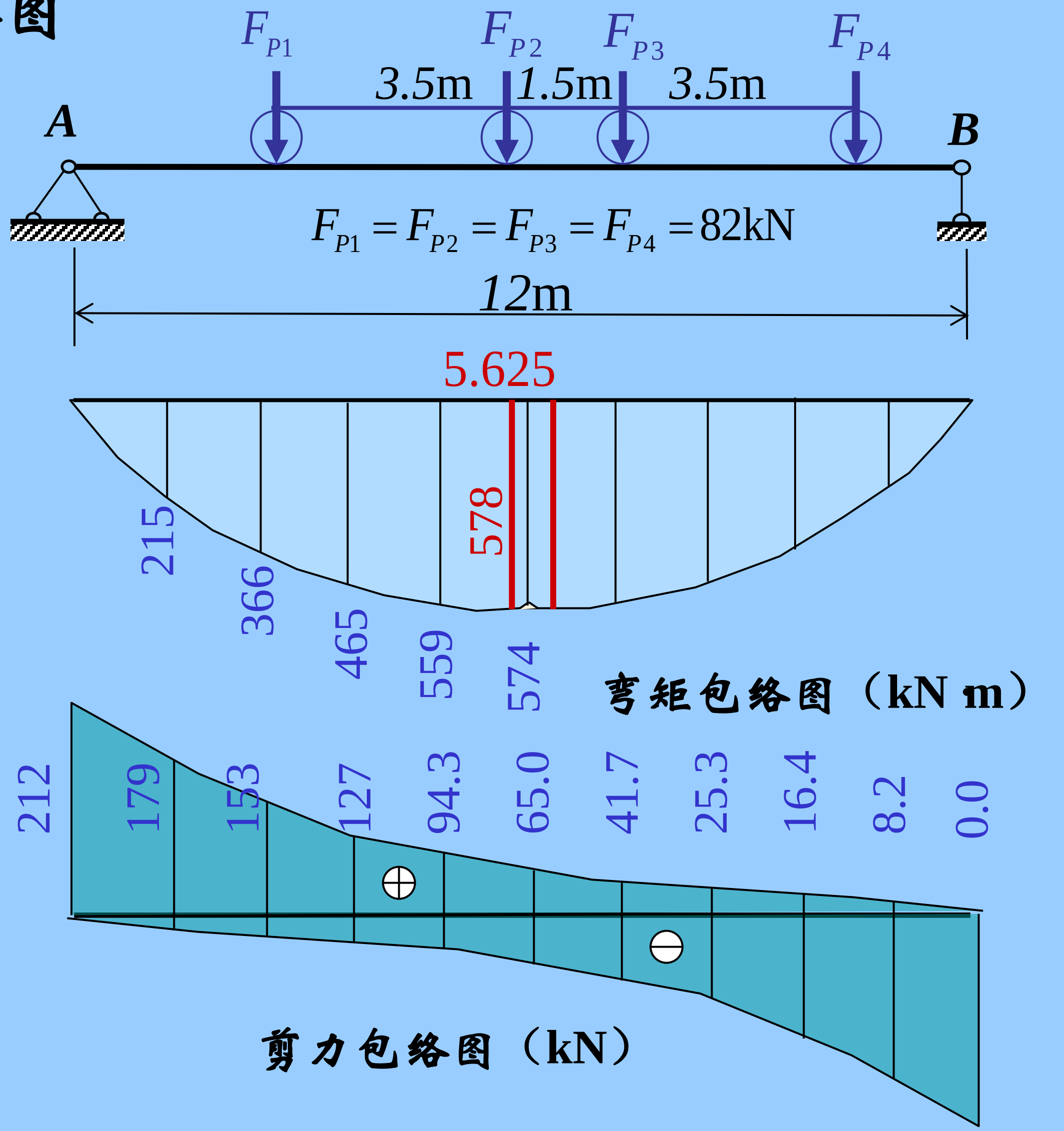
$$x=\frac{l-a}{2}=5.625\text{m}$$

$$M_{2\max}=(4\times82)\times5.625^2\times\frac{1}{12}$$

$$-(82\times3.5)$$

$$=578\text{kN}\cdot\text{m}$$

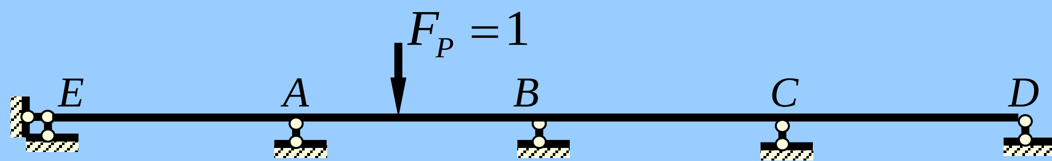
五 简支梁的内力包络图



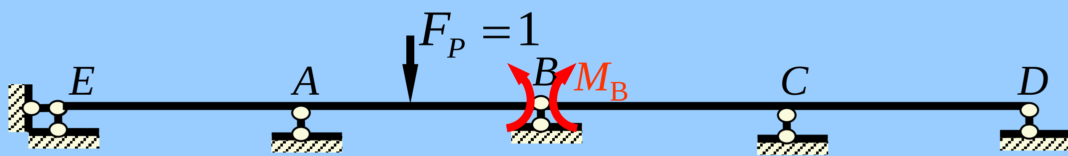
内力包络图：

将梁各截面的最大和最小内力按同一比例标在图上，分别连成的曲线。

六 超静定梁在可动均布荷载下的最不利荷载

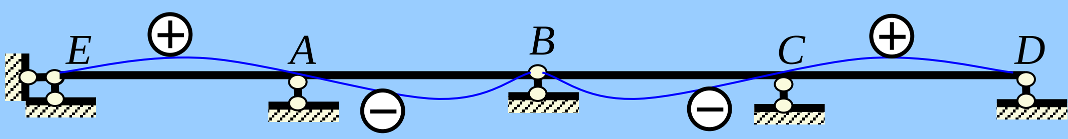


截面B弯矩影响线



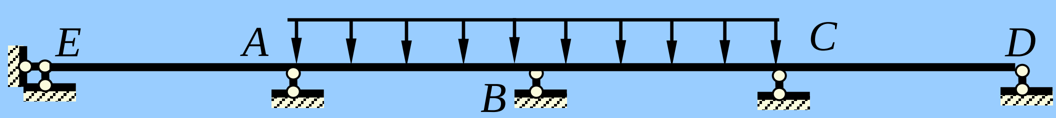
力法求位移和位移互等定理

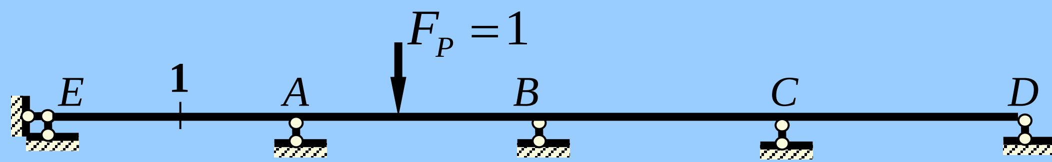
$$\Delta\varphi_B=\delta_{BB}M_B+\delta_{BP}=0$$



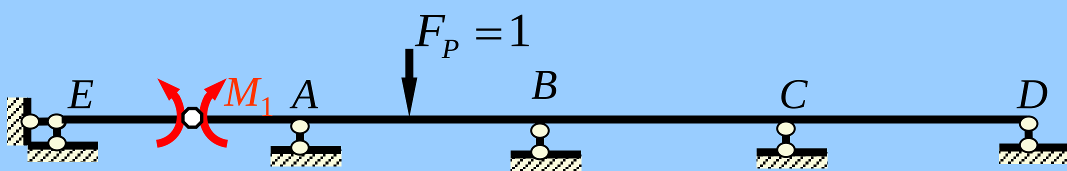
$$M_B=-\frac{\delta_{BP}}{\delta_{BB}}=-\frac{\delta_{PB}}{\delta_{BB}}$$

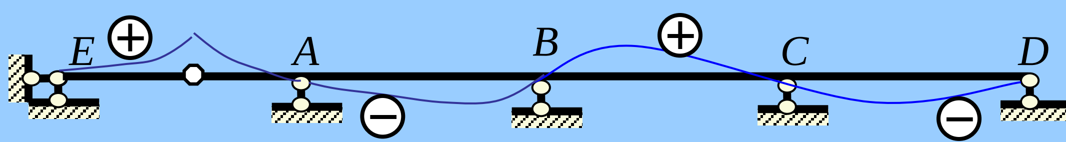
实际变形曲线轮廓　影响线轮廓



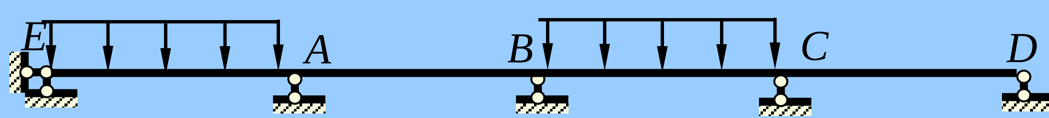


截面1最大正弯矩

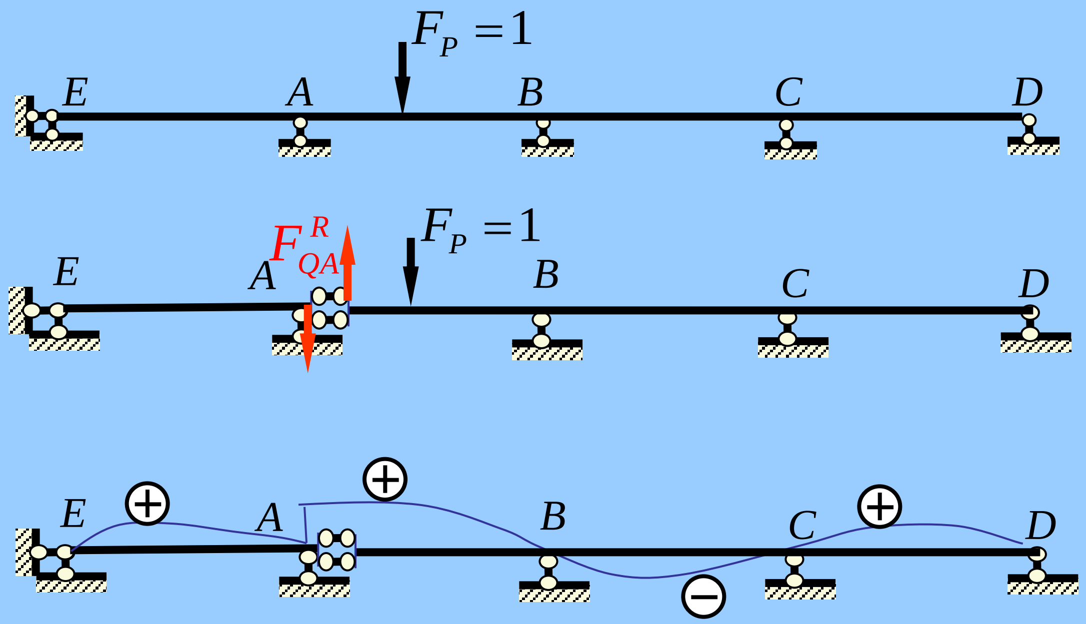




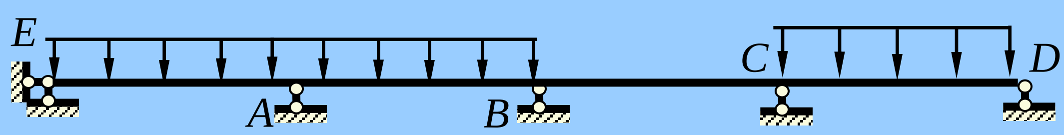
实际变形曲线轮廓　影响线轮廓



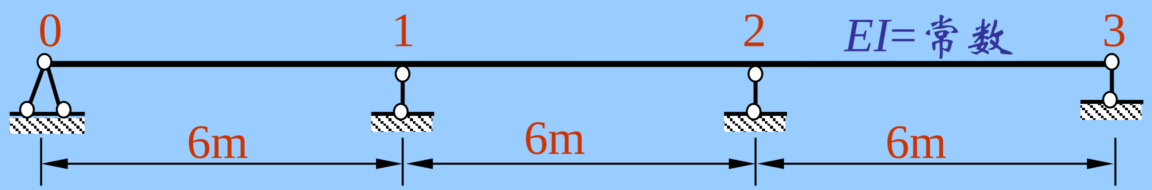
A右截面最大正剪力

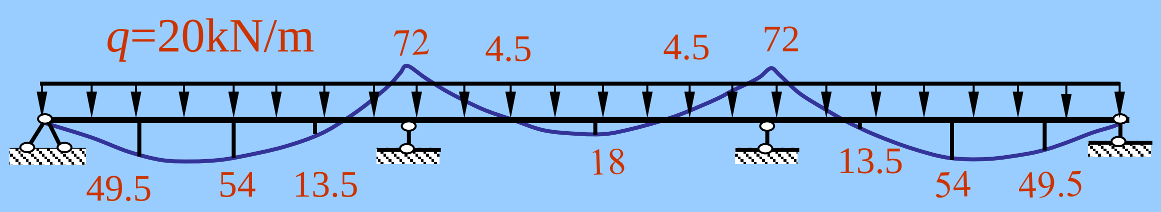


实际变形曲线轮廓　影响线轮廓

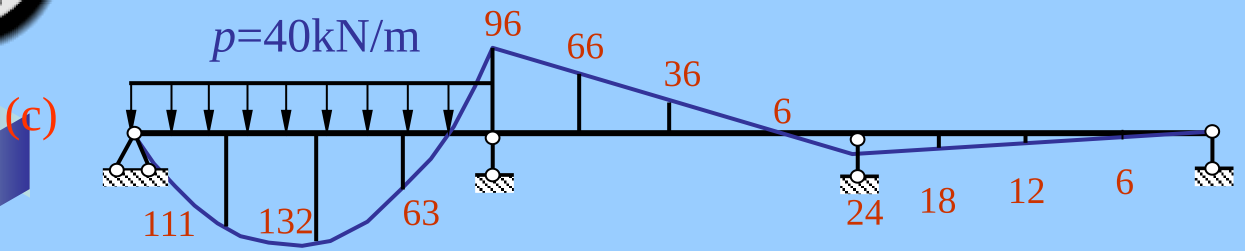


例 **试绘制图示三跨等截面连续梁的弯矩包络图。梁上承受的恒载为$q = 20$kN/m，均布活载为$p = 40$kN/m。**

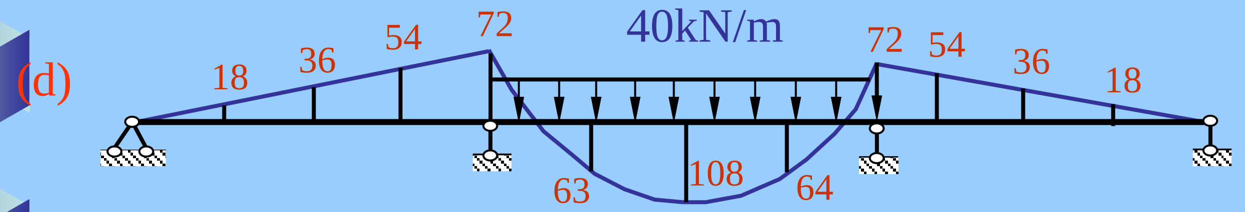




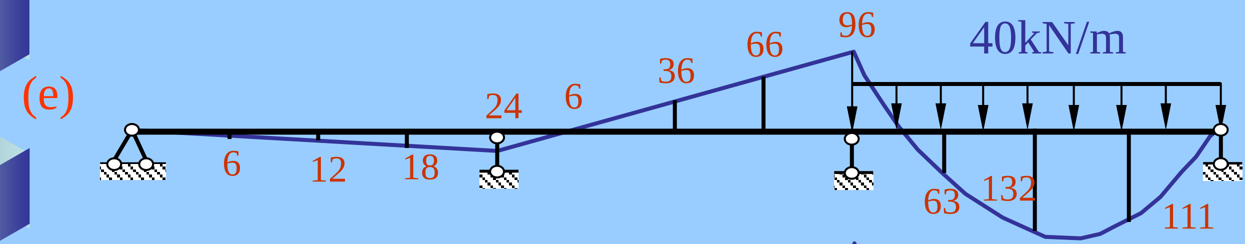
恒载的*M*图 (kN m)



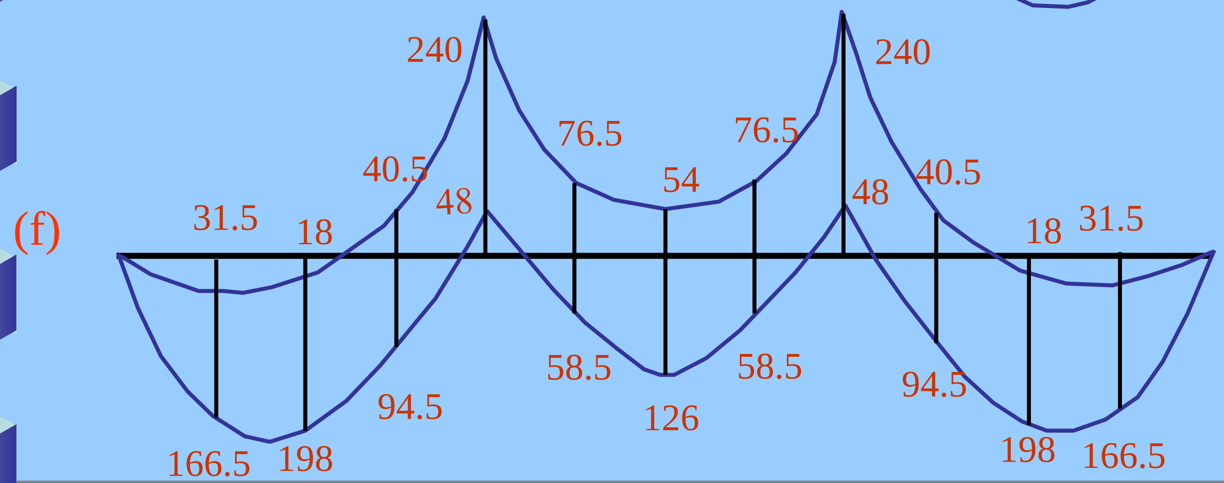
活载在第一跨的M图 (kN m)



活载在第二跨的M图 (kN m)



活载在第三跨的M图 (kN m)



弯矩包络图
(kN m)